A Regression Analysis of Pilot-Induced Oscillation Ratings

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Pilot-induced oscillation (PIO) has been studied by fitting a regression surface to flight test data published by the Air Force Flight Dynamics Laboratory. The independent variables, characteristics of the pitch rate step-response, include time to first peak, effective time delay, slope after first peak, and stick force per *g*. These were selected mainly from theoretical considerations and partially from their fit in the regression analysis. These variables were correlated to the PIO numerical ratings, and least-square regression surfaces were calculated. Various models were considered, including logarithmic, linear, and second-order models. The results of this study are a set of parameters that can be measured on the time history of the pitch rate step response, and an ordering of these parameters according to their relative importance (correlation) to PIO ratings. On the basis of this analysis, a complete simulation study is planned to investigate further the connection of these parameters with pilot ratings.

Nomenclature

c	= inverse of the radius of curvature of the first peak of the step response
d	= delay time measured from the intercept of the time axis and a line fitted through the rise to the first peak of the step response; effective time delay
F_{ES}	= force deflecting the elevator stick
ID	= index of determination
K_i	= i = 0,1,2,3 . . . const
m	= negative slope of a line fitted to the step response for about $\frac{3}{4}$ sec after the first peak
P	= PIO rating
$\hat{P}$	= estimated value of P
r	= radius of curvature estimated for the first peak of the step response
s	= stick force per g
S_P	= standard deviation of P measurements. Similarly S_{x_1}, S_{x_2} are standard deviation of x_1, x_2 measurements
t	= time to the first peak of the step response
β_i	= i = 1,2,3 . . . the Beta coefficients (see definition in text)
$\Delta d,\Delta m,\Delta s$ $\Delta t,\Delta P$	= increments of the variables $d,m,s,t,$ and P
θ	= pitch angle
$\left(\dfrac{\dot{\theta}}{F_{ES_{ss}}}\right)$	= steady-state value of pitch angle rate per unit stick force

Superscripts

$(\bar{\ })$	= average value
$(\dot{\ })$	= derivative taken with respect to time

Introduction

A current research effort at Grumman Aircraft correlates the handling qualities of a manned dynamic system with characteristics of the system's step response. An exploratory study of pilot-induced oscillation (PIO) in a simple, single-degree-of-freedom-pitch, visual-only tracking task has begun. In line with this effort, Air Force data reported in AFFDL-TR 68-90 (Ref. 1) were analyzed to determine how certain step response characteristics, deemed important through theoretical inquiry, really affect PIO ratings. This was to establish a starting point for the broader investigation. The Ref. 1 data were used since they were the most readily available flight test data that included separate PIO ratings and the step responses of each configuration evaluated.

The theoretical explanation of the cause of pilot-induced oscillation arose as a result of noticing an "echo" in simulation results involving an abruptly terminated impulse response. The simulation uses a real time convolution program to determine the output of a linear system and a time varying forcing function. This program is limited in its capability to represent the system's impulse response because it has a small computer memory. As a consequence of the small memory, an abruptly terminated impulse response was caused. It was noticed that this caused an echo and T. Keller of Grumman pointed out, it is this echo which may be a prime cause of pilot-induced oscillation.

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Theory

The echos which may be a prime cause of pilot-induced oscillation arise as follows. Consider Fig. 1; the step response and input function are idealized to demonstrate the effect.

Table 1 PIO tendency rating scale[1]

Description	Numerical rating
No tendency for pilot to induce undesirable motions.	1
Undesirable motions tend to occur when pilot initiates abrupt maneuvers or attempts tight control. These motions can be prevented or eliminated by pilot technique.	2
Undesirable motions easily induced when pilot initiates abrupt maneuvers or attempts tight control. These motions can be prevented or eliminated but only at sacrifice to task performance or through considerable pilot attention and effort.	3
Oscillations tend to develop when pilot initiates abrupt maneuvers or attempts tight control. Pilot must reduce gain or abandon task to recover.	4
Divergent oscillations tend to develop when pilot initiates abrupt maneuvers or attempts tight control. Pilot must open loop by releasing or freezing the stick.	5
Disturbance or normal pilot control may cause divergent oscillation. Pilot must open control loop by releasing or freezing the stick.	6

Beneath the step response is shown the corresponding impulse response (the derivative of the step response). The impulse response is abruptly terminated. The input function is taken as two opposite pulses. This type of input may result from the pilot, let us say, reacting to a wind gust. The input causes a triangular output and at time t_2 an echo occurs. The pilot, upon sensing the echo may construe it to be a new wind gust. He may then react to this and give a similar command, thus repeating the cycle. The reaction would produce oscillations, and due to different gains these oscillations may even diverge. The factors involved in this phenomenon include the time delay and the time to the first peak. The slope of the line following the peak will determine the gain and govern the size of the echo. The curvature of the first peak acts in a similar manner. It is also suggested in Ref. 1 that stick force per g may be another important factor in PIO.

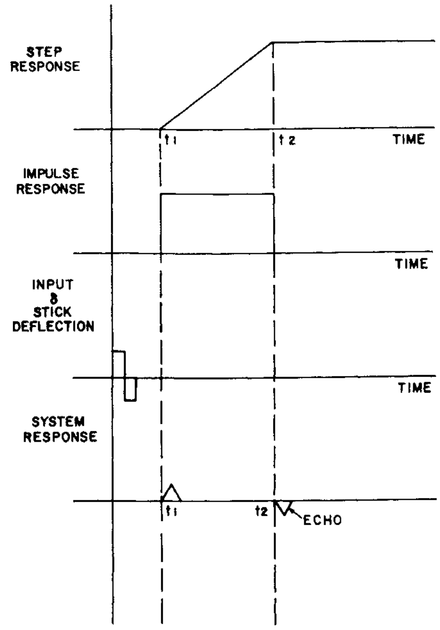

Fig. 1 Idealized explanation of PIO.

Regression Models

The following parameters were considered for theoretical reasons to be potentially important factors in causing pilot-induced oscillation (PIO) (see Fig. 2): d = "effective" time delay, t = time to the first peak of the step response, m = negative slope of a line fitted after the first peak of the step response, c = inverse of radius of curvature of the first peak, and s = stick force per g.

The parameters were measured from the step response of pitch rate per unit stick force. Data comprising step responses and associated PIO ratings for flights with a variable stability T33 were published in Ref. 1. The basis for the PIO ratings is shown in Table 1. The description of the flights and experimental techniques is given in Ref. 1.

In all, 22 pieces of data are measured (see Table 2). The measured data were then studied to determine a least-squares best fit regression surface. Linear, logarithmic, and second-order models were considered. To compare results obtained with various models, the index of determination (ID), F ratio statistic, and mean-square of the remainder were calculated for each model. The index of determination is defined as the ratio of the regression sum squares to the total sum squares. The regression sum of squares (SSR) is

$$\mathrm{SSR} = \sum_{i=1}^{n} (\bar{P} - \hat{P}_i)^2 \tag{1}$$

where $\bar{P}$ is the average value of PIO, $\hat{P}_i$ is the estimated value of PIO, and n is the total number of measured points. The total sum of squares (SST) is

$$\mathrm{SST} = \sum_{i=1}^{n} (P_i - \bar{P}_i)^2 \tag{2}$$

where $\bar{P}$ is the average value of PIO of the measured data and P_i is the measured value of PIO. The F ratio statistic is the ratio of the mean square regression (MSR) to the mean square error (MSE), where MSR = SSR/K and K is the degrees of freedom in the regression, and where MSE = (SST − SSR)/m, where m is the remaining degrees of freedom (total degrees of freedom − 1 − K). The GE Mark I computer (program MREG1***) was used to calculate the regression coefficients and measures.

Part I was an exploratory investigation preparatory to a more complete, in-depth study. Only part of the data was used, and the purpose was to find a direction, to determine whether a log fit or a linear-type fit was more suited to the data, and to decide whether the measures made on the step response were appropriate. Part II used all the data available and included some new measures. The measure c (the inverse of the radius of curvature) was discarded in favor of m (the negative of the slope after the first peak) because c is difficult to measure accurately and m appeared from theoretical considerations to be more closely related to a possible cause of

Table 2 PIO data from Ref. 1

Page	Run no.	P	t	m	s	d	c
125A	862	3.0	1.10	1.14	16.8	0.30	...
123A	841	1.0	1.00	1.00	8.39	0.25	...
119A	863	3.0	0.90	0.80	7.90	0.28	...
117A	848	1.5	0.90	0.62	6.81	0.20	...
115A	860	4.5	1.25	1.00	25.2	0.48	...
113A	837	2.0	0.90	0.80	8.39	0.28	...
109A	846	4.5	1.05	0.75	6.81	0.25	...
107A	847	1.5	0.80	0.60	8.39	0.16	...
105A	852	1.0	0.75	0.66	4.19	0.10	...
155C	857	3.5	0.55	3.7	7.96	0.16	6.66
153C	866	4.5	1.20	2.4	24.2	0.55	2.22
151C	864	4.0	1.10	2.9	23.8	0.50	3.33
149C	854	4.5	0.65	3.4	11.9	0.20	1.54
147C	844	4.0	0.60	2.9	8.05	0.21	1.67
145C	835	3.0	0.70	4.1	8.14	0.24	1.43
141B	848	4.5	1.10	2.1	10.2	0.24	0.91
139B	864	6.0	1.30	2.3	9.79	0.60	0.77
137B	851	4.5	0.90	2.1	9.54	0.28	1.11
135B	838	3.0	0.87	2.2	9.79	0.21	1.15
133B	839	4.0	1.10	1.8	10.4	0.21	0.91
129A	857	1.0	0.75	2.1	10.2	0.08	1.33
127A	865	4.5	1.2	0.80	8.39	0.39	0.83

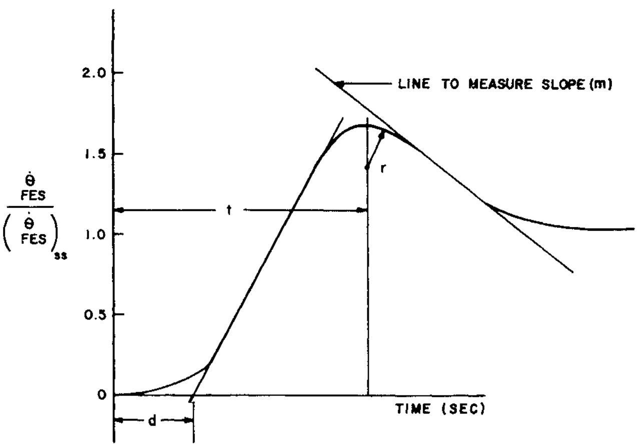

Fig. 2 Normalized rate of pitch per unit stick force step response.

Table 3 Model measures

	ID	F statistics
Model A	0.3029	2.17
Model B	0.1634	0.976

PIO. In Part II, various models were tried until the final result was achieved.

Part I—Exploratory Inquiry

In Part I, the last 13 pieces of data in Table 2 were analyzed to determine the coefficients for the models,

Model A

$$\hat{P} = K_1 + K_2 t + K_3 c \quad (3)$$

Model B

$$\hat{P} = K_1 t^{K_2} c^{K_3} \text{ or } \log\hat{P} = \log K_1 + K_2 \log t + K_3 \log c \quad (4)$$

where $\hat{P}$ is the PIO rating estimated by regression analysis and K represents the constants to be computed by regression techniques. The results obtained were

Model A

$$\hat{P} = 1.28 + 2.67t + 0.0742c \quad (5)$$

Model B

$$\hat{P} = 3.85t^{0.663}c^{0.0836} \quad (6)$$

The indexes of determination and the F ratio test statistics for the two models are shown in Table 3.

The superiority of the linear fit ($ID = 0.30$) over the log fit ($ID = 0.16$) led us to consider additional parameters in a linear and second-order model in Part II. It is apparent from the low ID's, that neither of the fits in Part I was particularly good. The F ratio statistic does not even reach the 95% level of significance. Consequently, in Part II new measures of the step response were added to the model.

Part II—Fitting the Model

The data for Part II were expanded to 22 points from the 13 of Part I, and additional variables were considered. Instead of the parameter c, the parameter m was chosen to reflect the effects on PIO at the first peak of the step response. Thus, the parameters were t, m, tm, and s. We attempted to fit

$$\hat{P} = \bar{P} + K_1(t - \bar{t}) + K_2(m - \bar{m}) + K_3(tm - \overline{tm}) + K_4(s - \bar{s}) \quad (7)$$

where the mean values of $\bar{P}$, $\bar{t}$, $\bar{m}$, $\overline{tm}$, and $\bar{s}$ were 3.32, 0.94, 1.82, 1.63, and 11.1, respectively. The results are shown in Table 4.

These results were encouraging; they showed an index of determination of 0.63 for the full model and 99% significant F ratio statistics for the full and reduced models. At this point in the analysis, the effective time delay d was added to the list of parameters. The new results with this variable added (see Table 5) show an index of determination improvement from 0.63 to 0.67.

In the next level of inquiry, a second-order model in t, m, and s was studied. Table 6 shows the results. The significance of the 0.71 index of determination for the full model is somewhat reduced since the 22 pieces of data in this case were used to determine 9 coefficients. This degradation of significance is reflected by the fact that the F ratio statistic fell below the 99% level.

Trial E in Table 6 indicates that with five variables m, s, tm, and t^2, we have an ID of 0.67, have reduced the mean square error from 2.06 to 0.944, and have an F statistic of >99% significance. This seemed very promising and indicated the next step, to combine effective time delay d with these five variables to see if this model would enhance the fit. The results appear in Table 7. Since Trial D seemed very promising, Trials E and F were made, deleting datum No. 841 (Table 2), which seemed in each run to be far out of fit compared to the balance of the data. Trial F with all variables included was not good as Trial E, as indicated by a poorer mean square error.

In Table 8 are shown the estimated and measured values of P and the error and percent error for Trial E. Trial E of Table 7 represents the best fit of the entire study, using the fewest variables with the most significance. Trial E of Table 7 yields the equation

$$\hat{P} = 3.43 + 1.85(m - 1.87) - 1.44(tm - 1.66) + 3.50(d - 0.282) + 3.99(t^2 - 0.924) - 0.0618(ts - 11.2) \quad (8)$$

This reduces to

$$\hat{P} = -1.63 + 1.85m + 3.50d - 1.44tm - 0.0618ts + 3.99t^2 \quad (9)$$

One further point must be noted concerning the effect of deleting the datum point no. 841 from the E and F regression fits in Table 7. Comparing D and E, we see that the coefficients are not much affected by deleting the one datum point. Since we are concerned with the relative significance of the variables, the conclusions of the analysis do not change; all that is involved is the choice as to whether we have a 0.68 index of determination with all data points or a 0.75 index of determination with one point deleted.

Analysis of Results

Having obtained a reasonable 5-term regression function for $\hat{P}$, we proceed to analyze the meaning of the variables in-

Table 4 Results of first regression model

Trial	Coefficients of the variables				Index of determination	F ratio statistic[a]	Mean square of remainder (Total = 2.06)
	t	m	tm	s			
A	9.07	2.33	−1.65	−0.0396	0.63	7.32†	0.935
B	5.43	0.950	...	−0.0413	0.58	8.26†	1.01
C	8.39	2.26	−1.67	...	0.62	9.68†	0.920
D	2.83	...	0.949	−0.0281	0.49	5.79†	1.22
E	4.67	0.856	...	...	0.56	12.2†	0.996
F	2.48	...	0.875	...	0.48	8.89†	1.18
G	...	...	0.966	...	0.35	10.56†	1.41
H	3.00	...	...	...	0.21	5.19*	1.72
I	...	0.493	...	...	0.14	3.23	1.86
J	...	...	0.871	0.310	0.36	5.30*	1.46
K	...	−0.449	1.40	0.00809	0.39	3.80*	1.47

[a] Note: * = >95%, † = >99%.

Table 5 Results of second regression model

Trial	Coefficients of the variables t	m	tm	s	d	Index of determination	F ratio statistic[a]	Mean square of remainder (Total = 2.06)
A	6.92	2.16	−1.66	−0.0628	4.32	0.67	6.60†	0.883
B	6.63	2.11	−1.69	...	2.99	0.64	7.53†	0.918
C	...	0.437	0.0802	−0.0544	7.83	0.57	5.56†	1.10
D	2.91	0.701	...	...	2.93	0.58	8.42†	1.00
E	...	0.452	...	...	6.52	0.54	11.2†	1.05
F	...	0.457	−0.00811	...	6.54	0.54	7.07†	1.10

[a] Note: † = >99%.

Table 6 Results of third regression model

Trial	Coefficients of the variables t	m	s	tm	ts	ms	t^2	s^2	m^2	Index of determination	F ratio statistic[a]	Mean square of remainder (Total = 2.06)
A	−1.18	3.72	0.233	−1.70	−0.728	−0.0168	8.69	0.0185	−0.337	0.71	3.23*	1.05
B	−21.4	...	0.404	0.461	−0.386	−0.00336	15.2	...	...	0.64	4.49†	1.03
C	−23.0	...	0.365	...	−0.396	0.0336	16.5	...	...	0.63	5.53†	0.991
D	−21.3	...	0.415	0.501	−0.390	...	15.1	...	...	0.64	5.74†	0.968
E	−9.05	1.37	0.271	−0.878	−0.269	...	9.82	...	...	0.67	5.13†	0.944

[a] Note: * = >95%, † = >99%.

volved. Four steps are taken. Determine the beta (β_i) coefficients, i.e., find the sensitivity of the function to the respective terms. Determine the partial derivatives and evaluate them at the mean values of the variables. Using the partials obtained in step two, apply reasonable step sizes of the variables to determine their respective effects in causing changes in $\hat{P}$. Perform an analysis of regression fit models term by term to array the variables according to their improvement of the F statistic.

It is important to note that no conclusions are drawn concerning the causal relationship between the equation result and the PIO phenomenon. The analysis is intended as a guide to determine the significant variables and their relationships. The question of causality requires further study.

It is necessary to consider how the tool of regression analysis is applied to the problem at hand. The PIO rating scale is, in a sense, partly ordinal and partly interval. It is ordinal because it does not purport to represent a given degree of degradation from number to number. But it is interval in the sense that intuitively one feels that the ratings proceed from one to six indicating progressively worse systems. However, the interval from number to number may be uneven. We don't know the interval because we have no absolute common measure to relate all the numbers. This regression analysis proceeds as if there were some regularity in the interval from rating to rating. The "goodness of fit" obtained would indicate that this assumption is valid. In fact the table of PIO ratings could now be revised slightly to correspond more closely to the results of the regression analysis. However, this is not the point. What is being sought here is an ordering of factors that contribute to PIO. A strong indication derives from this study that future investigation should proceed by studying the indicated step response parameters. These appear to have a direct connection with the actual phenomenon of PIO. In Ref. 2, some discussion is given of treating ordinal scales as if they were interval scales.

Beta Coefficients

Beta (β) coefficients are the weighted values of the regression coefficients such that they all have a mean of zero and a standard deviation of 1. This is equivalent to solving the equation

$$\frac{\hat{P} - \bar{P}}{S_P} = \beta_1 \left(\frac{x_1 - \bar{x}_1}{S_{X_1}}\right) + \beta_2 \left(\frac{x_2 - \bar{x}_2}{S_{X_2}}\right) \tag{10}$$

where $S_P, S_{X1}, S_{X2}, \ldots$ are the standard deviations of the subscripted variable.

To determine the respective beta coefficients each regression coefficient is multiplied by a factor such as (for m)

$$\beta_m = b_m \left(\frac{S_m}{S_P}\right) \tag{11}$$

where b_m is the coefficient for the m term (1.85), S_m is the standard deviation of m, and S_P is the standard deviation of P. The β coefficients are

$$\beta_m = 1.47; \quad \beta_d = 0.356; \quad \beta_{tm} = -0.927; \quad \beta_{ts} = -0.367; \quad \beta_{t2} = 1.21$$

Table 7 Results of final regression model

Trial	Coefficients of the variables t	m	tm	d	t^2	ts	Index of determination	F ratio statistic[a]	Mean square of of remainder (Total = 2.06)
A	−1.71	1.75	−1.26	3.51	4.57	−0.0540	0.69	5.43*	0.908
B	−17.7	...	0.594	2.68	10.9	−0.0489	0.63	5.50*	0.995
C	−6.54	1.58	−0.984	...	7.64	−0.0387	0.66	6.32*	0.909
D	...	1.84	−0.136	3.65	3.76	−0.0546	0.68	6.95*	0.852
									(Total = 1.88)
E[b]	...	1.85	−1.44	3.50	3.99	−0.0618	0.75	8.96*	0.629
F[b]	4.94	2.12	−1.71	3.90	1.63	−0.0637	0.75	7.07*	0.667

[a] Note: * = >99%.
[b] One piece of data was deleted, run no. 841, Table 2.

Table 8 Trial E tabulation

Calculated	Observed	Difference	%
3.42	3.00	0.427	12.4
2.59	3.00	−0.400	−15.4
2.27	1.50	0.777	34.1
4.39	4.50	−0.102	−2.32
2.57	2.00	0.572	22.2
3.45	4.50	−1.04	−30.2
1.50	1.50	0.273E-02	0.18
1.27	1.00	0.276	21.6
3.85	3.50	0.358	9.30
4.52	4.50	0.225E-01	0.50
4.10	4.00	0.102	2.49
3.42	4.50	−1.07	−31.4
3.18	4.00	−0.816	−25.6
4.24	3.00	1.24	29.3
3.93	4.50	−0.567	−14.4
6.37	6.00	0.372	5.85
3.21	4.50	−1.28	−39.8
2.95	3.00	−0.492E-01	−1.67
3.69	4.00	−0.309	−8.39
2.02	1.00	1.02	50.5
4.96	4.50	0.466	9.40

This indicates the order m, t^2, tm, ts, d; most significant to least significant.

Partial Derivatives

While the beta coefficient ordering is interesting, one cannot be sure of the basic variables t, d, s, and m, since the answer is in second-order terms. To discover the effect of the basic variables, we take partial derivatives of $\hat{P}$ with respect to each variable, in the equation for $\hat{P}$.

$$\begin{aligned}\partial\hat{P}/\partial t &= -1.44m + 7.98t - 0.0618s \\ \partial\hat{P}/\partial d &= 3.50 \\ \partial\hat{P}/\partial m &= 1.85 - 1.44t \\ \partial\hat{P}/\partial s &= -0.0618t\end{aligned} \tag{12}$$

We can now investigate the range of values of these partials or their sensitivity at the means of the other variables.

$$\begin{aligned}&\left.\frac{\partial\hat{P}}{\partial t}\right|_{\substack{m=\bar{m}\\ s=\bar{s}}} = -3.39 + 7.98t \quad \text{at } t = \bar{t} \quad \left.\frac{\partial\hat{P}}{\partial t}\right|_{\substack{m=\bar{m}\\ s=\bar{s}\\ t=\bar{t}}} = 4.07 \\ &\frac{\partial\hat{P}}{\partial d} = 3.50, \text{ a const} \\ &\left.\frac{\partial\hat{P}}{\partial m}\right|_{t=\bar{t}} = 0.50, \quad \left.\frac{\partial\hat{P}}{\partial s}\right|_{t=\bar{t}} = -0.06\end{aligned} \tag{13}$$

This shows a size order of partials at the means of

$$\frac{\partial\hat{P}}{\partial t} = 4.07; \quad \frac{\partial\hat{P}}{\partial d} = 3.50; \quad \frac{\partial\hat{P}}{\partial m} = 0.50; \quad \frac{\partial\hat{P}}{\partial s} = -0.06 \tag{14}$$

Changes in $\hat{P}$

To evaluate the contribution of each variable to the total $\hat{P}$ rating, it is useful to normalize the partial derivatives. There are a number of ways to do this. One such way is exemplified by the beta coefficients. An intuitively physical way is to decide upon a reasonable increment of each variable (Δv) and compute the increment in P,

$$\Delta P = \left|\frac{\partial\hat{P}}{\partial v}\right| \Delta v \tag{15}$$

Table 9 Criterion

β coefficient	F statistic	Absolute value of the partial derivative evaluated at the mean	ΔP: $\frac{1}{10}$ range variable size	ΔP: Standard deviation
m	d	t	t	t
t^2	m	d	d	m
tm	md	m	m	d
ts	t^2	s	s	s
d	ts			

A Δv was selected for each variable of one-tenth the range of the variable measured in the 22 data points. This resulted in $\Delta t = 0.075$, $\Delta m = 0.35$, $\Delta d = 0.052$, and $\Delta s = 2.1$. The resulting "normalized" values of ΔP are

$$\begin{aligned}\Delta P\Big|_{\substack{\text{due to } t\\ m=\bar{m}\\ s=\bar{s}\\ t=\bar{t}}} &= 0.306 \\ \Delta P\Big|_{\text{due to } d} &= 0.182 \\ \Delta P\Big|_{\substack{\text{due to } m\\ t=\bar{t}}} &= 0.175 \\ \Delta P\Big|_{\substack{\text{due to } s\\ t=\bar{t}}} &= 0.126\end{aligned} \tag{16}$$

This yields an ordering of t, d, m, and s.

An alternative step size (Δv) is the standard deviation of each variable. This yields $\Delta t = 0.222$, $\Delta m = 1.10$, $\Delta d = 0.143$, $\Delta s = 5.87$, and

$$\begin{aligned}\Delta P\Big|_{\substack{\text{due to } t\\ m=\bar{m}\\ s=\bar{s}\\ t=\bar{t}}} &= 0.90 \\ \Delta P\Big|_{\text{due to } d} &= 0.50 \\ \Delta P\Big|_{\substack{\text{due to } m\\ t=\bar{t}}} &= 0.55 \\ \Delta P\Big|_{\substack{\text{due to } s\\ t=\bar{t}}} &= 0.35\end{aligned} \tag{17}$$

This yields an ordering of t, m, d, and s. Note that m and d are very close to one another in significance.

F Statistic Order

Using a separate computer program (MULTI), we performed a regression analysis that arrays the input variables in order of their contribution to the fit by the F statistic criterion. The order was found to be d, m, md, t^2, and ts. This result is not in contradiction to the beta coefficients whose order is based on the sensitivity of each term.

Conclusion

In this analysis of data presented in Ref. 1 to find the sensitive parameters correlated with PIO ratings, four variables have been selected and a 0.75 index of determination has been found for the fit,

$$\hat{P} = -1.63 + 1.85m + 3.50d - 1.44tm - 0.0618ts + 3.99t^2 \quad (18)$$

Analysis of the contribution of the parameters t, d, m, and s has disclosed an ordering in their contribution to $\hat{P}$. The ordering is shown in Table 9. The purpose of this mathematical analysis was to discover the primary variables and their relative importance in relating to PIO. On the basis of the results, a complete simulation study is planned to gain further insight into the relationship between PIO and the characteristics of the step response.

References

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Flutter Induced by Aerodynamic Interference Between Wing and Tail

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Several authors have mentioned that aerodynamic interference between horizontal tail and wing could cause flutter. The unsteady aerodynamic forces pose the problem in analysis and have become available only recently. Naturally, the importance of interaction between two lifting-surface increases when both spans are of comparable magnitude. Several advanced aircraft feature this concept. A flutter analysis based on a variable-geometry airplane at highly swept condition is performed. The unsteady loads resulting from aerodynamic interference are introduced. They were determined with a lifting-surface method previously formulated by the coauthor. Destructive flutter was discovered resulting entirely from interference not predictable by applying conventional three-dimensional coefficients for the separate surfaces only. This verified a flutter case of a preceding experiment. The investigation is extended to include different vertical and aft positions of the horizontal tail at several Mach numbers to obtain an understanding of dominating influences.

Introduction

TWO years ago, Topp et al.[9] referred to an antisymmetric flutter case as experienced in a wind-tunnel test of an aircraft with variable wing geometry; see Fig. 1. This condition could not be determined theoretically by using conventional flutter approaches. After careful consideration of all prevailing influences, the instability was assumed to be due to the aerodynamic interference of wing and tail. No proof was given for this assumption, since at that time no method for the calculation of unsteady interference air loads was available, though H. Ashley had stimulated investigations in this field by a basic paper[1] as early as 1964.

In the meantime, B. Laschka and H. Schmid[6,10] investigated the aerodynamic interaction between oscillating wings and tails applying lifting-surface kernel function methods. It has been shown there that interference can be of considerable influence not only in the case of close proximity but also for large distances between wing and tail. Another approach by V. J. E. Stark[2] is based on lifting line lattices. Application has been reported by H. Wittmeyer[11] for the SAAB 37 Viggen flutter analysis. A similar procedure has also been proposed by E. Albano and W. Rodden.[3]

The amplitudes of the wing oscillation are usually small in the region near the body and high near the wing tip. Thus, high induced-velocity components usually can be expected downstream from the wing tip. Consequently, interference in most cases will become important where the span of the tail is not small relative to the wing span. Configurations showing these features are found on many modern airplanes especially on those with variable-geometry wings, or canard types. Conventional flutter analysis took into account the natural modes, generalized masses, and stiffnesses of the entire aircraft and applied the nonsteady airloads to wing and tail without interference effects. In the present paper, these interference effects no longer are neglected.

Flutter behavior on the variable geometry (VG) airplane configuration mentioned in Ref. 9 has been studied for the most critical case where the wing had a leading edge (LE) sweep of 70°. Theoretical analysis indicates a destructive flutter case resulting entirely from aerodynamic interaction that could not be predicted by applying only conventional three-dimensional aerodynamic coefficients to wing and horizontal tail separately. The experimentally experienced flutter case[9] thus could be verified. This result is of considerable interest, since before this investigation the question was raised as to

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